J Syst Sci Complex (2021) 34: 860–872

The Dynamics Characteristics of Flexible Spacecraft and Its Closed-Loop Stability with Passive Control*

MENG Bin · ZHAO Yunbo

DOI: 10.1007/s11424-020-9268-8
Received: 24 September 2019 / Revised: 27 December 2019
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Abstract Passive control is the most popular methodology for flexible spacecraft while it remains an open problem whether the closed-loop performance can be achieved only with passive control subject to the coupling modes of rigid and flexibility. Also, the closed-loop performance of passive PD control based on the dynamics of the Euler angle parameterization of spacecraft, which has been widely used in practice, is yet to be addressed. Towards these challenges, by introducing the input-output exact linearization theory and Lyapunov theory, the authors show that the closed-loop performance for flexible spacecraft with rigid and flexible modes can be achieved by adjusting the parameters of the passive controllers sufficiently large. This is done by firstly transforming the flexible spacecraft dynamics into an exact feedback linearization standard form, and then analyzing the closed-loop performance of flexible spacecraft.

Keywords Closed-loop stability, flexible spacecraft, internal dynamics, input-output exact linearization, passive control.

1 Introduction

Increasing tasks performed by spacecraft are recently making use of flexible appendages including, e.g., solar arrays, large scale of antennas, stellar objects, etc. These appendages are often of large scale, but are usually made from light novel material to diminish launch mass. This thus enables the flexible structure of modern spacecraft, and consequently how to deal with this structural flexibility has become an important and difficult problem in the field of space control. Considerable works have already been found in the literature, especially since 1990s (see [1–5]).

MENG Bin
Beijing Institute of Control Engineering, Beijing 100094*, China; Science and Technology on Space Intelligent Control Laboratory, Beijing* 100094*, China.* Email: mengb@amss.ac.cn.
ZHAO Yunbo
Zhejiang University of Technology, Hangzhou 310014*, China.*
*This paper was supported by the National Key RδD Program of China under Grant No. 2018YFA0703800, the Science and Technology on Space Intelligent Control Laboratory Foundation of China under Grant No. ZDSYS-2018-04, and the National Natural Science Foundation of China under Grant Nos. 51805025 and 61673350.
◇ *This paper was recommended for publication by Editor-in-Chief CHEN Jie.*

As a high order nonlinear system with the states being the spacecraft attitudes, the angular velocities, and the flexible modes, the dynamics of flexible spacecraft and its control design can be categorized as either active or passive, whose difference relies on whether the flexible modes need to be measured or not. Active control makes use of the intelligent structures to measure flexible modes and designs whole state feedback with flexible measurement. Robust control and adaptive control are two often employed methodologies in this regard (see [6]). However, active control may often be infeasible in practice since the flexible modes are often difficult, if not impossible, to measure. This difficulty thus makes passive control methods popular in practice, where only the rigid body information but not the flexible modes measurement is needed. As is known the dynamics of flexible spacecraft is a high order nonlinear system, while passive control only applies to the rigid states. This then leads to an immediate problem which is fundamental to flexible spacecraft: Is the spacecraft stabilizable under low order control? Towards this problem progress has already been made to a certain extent. For single axis spacecraft dynamics, the closed-loop stability is proved under passive PD control in [6]. Similar stability criteria, without tracking, have also been given for three axes kinematics in terms of quaternion parameterization with the same feedback coefficient for the three axes in [7], but unfortunately not directly applicable to the case where the feedback coefficients for different axes are different, due to the specific Lyapunov function used in the proof. The kinematics of spacecraft in terms of quaternion parameterization is algebraic without triangular functions, which is a popular model in flexible spacecraft (see [7–9]). Furthermore, it is widely accepted in practice to design passive control for the kinematics and dynamics using the Euler angle parameterization because their physical meaning is very specific. However, the dynamics is strong nonlinear with triangular function (see [10]), resulting in a very complex closed-loop system, making the performance analysis of the closed-loop system still an open problem.

Following this research line in this work we investigate the stabilization and tracking problems for flexible spacecraft using the passive PD control, with the aim at laying the theoretical foundation for the practical applications of the passive PD control for flexible spacecraft. The remainder of the work is organized as follows. We formulate the problem of interest in Section 2, and then present the main results in Section 3 and finally conclude the paper in Section 4.

2 Preliminaries and Problem Formulation

We first introduce the theory of nonlinear input and output linearization in order to study the properties of passive PD control for flexible spacecraft.

2.1 Input-output Exact Linearization (see [11])

Consider the following affine nonlinear system:

$$\begin{cases} \dot{x} = F(x) + G(x)u, \\ y = h(x), \end{cases} \tag{1}$$

where F, G and h are smooth enough functions in a domain $\mathcal{D}$, and x, y and u are the states, outputs and control inputs, respectively.

If the relative degree of the nonlinear system (1) is well defined, via a diffeomorphism transformation it can be transformed into an exact feedback linearization standard form consisting of two subsystems called the interior and exterior dynamics, respectively, as follows,

$$\begin{cases} \dot{\xi} = A\xi + B\gamma(x)(u - \alpha(x)), \\ \dot{\eta} = f_0(\xi, \eta), \end{cases} \tag{2}$$

where f_0, α and γ are continuous and differentiable, $f_0(0,0) = 0$, (A, B) is controllable, and $\gamma(x)$, $x \in \mathcal{D}$, is nonsingular.

2.2 Passive PD Control for Flexible Spacecraft in Practice

By omitting the orbit angular velocity, the dynamics of flexible spacecraft can be obtained as follows (see [10]):

$$\begin{aligned} &\left[\dot{\phi} \;\; \dot{\theta} \;\; \dot{\psi}\right]^{\mathrm{T}} = C(x_1)\left[w_x \;\; w_y \;\; w_z\right]^{\mathrm{T}}, \\ &\begin{cases} I_s\dot{w}_s + \widetilde{w}_s I_s w_s + F_{sL}\ddot{\eta}_L + F_{sR}\ddot{\eta}_R = T_s, \\ \ddot{\eta}_L + 2\xi_L w_L \dot{\eta}_L + w_L^2\eta_L + F_{sL}^{\mathrm{T}}\dot{w}_s = 0, \\ \ddot{\eta}_R + 2\xi_R w_R \dot{\eta}_R + w_R^2\eta_R + F_{sR}^{\mathrm{T}}\dot{w}_s = 0, \end{cases} \\ &y = x_1, \end{aligned} \tag{3}$$

where ϕ, θ, ψ are the roll, the pitch, and the yaw attitudes, w_x, w_y, w_z are the roll, the pitch and the yaw angle velocities, respectively,

$$x_1 = \left[\phi \;\; \theta \;\; \psi\right]^{\mathrm{T}}, \tag{4}$$

$$C(x_1) = \begin{bmatrix} \frac{\cos\theta}{\cos\psi} & 0 & \frac{\sin\theta}{\cos\psi} \\ \tan\psi\cos\theta & 1 & \tan\psi\sin\theta \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \tag{5}$$

is the transformation matrix,

$$w_s = \left[w_x \;\; w_y \;\; w_z\right]^{\mathrm{T}},$$

$$\widetilde{w}_s = \begin{bmatrix} 0 & -w_z & w_y \\ w_z & 0 & -w_x \\ -w_y & w_x & 0 \end{bmatrix}$$

represents the cross-product operator matrix, $\eta_L \in \mathcal{R}^l$ and $\eta_R \in \mathcal{R}^l$ are the modes coordinate matrices of the left and right solar arrays, $\xi_L > 0$ and $\xi_R > 0$ are the mode damping coefficients of the left and right solar arrays, $w_L > 0$ and $w_R > 0$ are the mode frequencies of the left and

right solar arrays, $F_{sL} \in \mathcal{R}^{3\times l}$ and $F_{sR} \in \mathcal{R}^{3\times l}$ are the coupled matrices between flexible and rigid bodies of the left and right solar arrays, T_s is the external torque vector acting on the spacecraft, and I_s is the inertia matrix of spacecraft. Here we consider the case that $0 \leq \psi < 90^{\rm o}$ (when $\psi \geq 90^{\rm o}$, the model of spacecraft in terms of quaternion parameterization needs to be introduced). Through simple computation, $C(x_1)$ is nonsingular when $0 \leq \psi < 90^{\rm o}$.

In practice, passive PD control has been a main control strategy for flexible spacecraft due to the difficulty in the measurement of flexible modes and the simple feasibility of the PD control. Passive control only makes use of rigid information and only exerts control for rigid components without making use of flexible information and exerting control for flexible components. The passive PD control is designed according to the following procedures in practice. Firstly, simplify the dynamics (3) by omitting $C(x_1)$ and the flexible components of (3), which results in the following simplified dynamics,

$$\begin{bmatrix}\dot{\phi} & \dot{\theta} & \dot{\psi}\end{bmatrix} = w_s, \quad \dot{w}_s = -I_s^{-1}\widetilde{w}_s I_s w_s + I_s^{-1}T_s. \tag{6}$$

Then, design the passive PD stabilization control according to (6),

$$T_s = \widetilde{w}_s I_s w_s + I_s(-K_P x_1 - K_D w_s), \tag{7}$$

which in essence is a feedback linearization based PD control and where both K_P and K_D are diagonal matrices and are expressed as

$$\begin{aligned} K_P &= \mathrm{diag}(k_{pi}), \quad k_{pi} > 0, \quad i = 1, 2, 3, \\ K_D &= \mathrm{diag}(k_{di}), \quad k_{di} > 0, \quad i = 1, 2, 3. \end{aligned} \tag{8}$$

It is easy to see that under the control of (7) the rigid component (6) is exponentially stable. In what follows we are going to prove the closed-loop performance of the flexible dynamics (3) under the control (7).

Remark 2.1 In [7], the passive PD stabilization performance has been shown for flexible spacecraft. However, the parameterizations of kinematics and the forms of the PD passive controls are different from this work, as follows:

• The parameterizations of the kinematics are different. In [7], the parameterization is in terms of quaternion, which is algebraic without triangular functions, while in this work the kinematics and dynamics are in terms of Euler angle, which has been extensively applied in practice due to its intuition. However, Euler angle parameterization includes triangular functions with strong nonlinearity, leading to the difficulties of analysis for closed-loop performance.

• The forms of the passive PD controls are different. In [7], the PD control has the same feedback coefficients for the three axes control, while in this work it has different coefficients for the three axes control. The design in [7] cannot be applied directly to our case because the Lyapunov function introduced in [7] is only feasible when the coefficients are the same for the three axes. Apparently, to realize control objectives of spacecraft, in general, different coefficients for three axes should be chosen.

• Unlike this work the tracking problem was not considered in [7].

We have the following assumption for the reference signal $r(t)$.

Assumption 1 The reference signal $r(t)$ and its derivatives $\dot{r}(t)$ and $\ddot{r}(t)$ are bounded.

Similar to the stabilization control (7), the tracking control is a combination of a feedback linearization based PD tracking control and a feedforward control,

$$T_s = \widetilde{w}_s I_s w_s + I_s \ddot{r} + I_s(-K_P(x_1 - r) - K_D(w_s - \dot{r})). \tag{9}$$

It is easy to see that under the control of (9), exponentially tracking can be achieved for the simplified dynamics (6). Similar to stabilization problems, we are going to prove the closed-loop performance of the flexible dynamics (3) under the control (9).

2.3 Problem Formulation

In this paper, we consider the passive control performance for the flexible dynamics of spacecraft. Specifically, for the flexible spacecraft dynamics (3), the closed-loop performances are studied under the control of the passive control (7) and (9).

3 Main Results

The main results are given in this section. Firstly, we transform the dynamics of spacecraft into the standard form (2). We then analyze the properties of the whole systems by analyzing that of subsystems. Lastly, we analyze the properties of the passive control (7) and (9).

3.1 Dynamics Transformation for Flexible Spacecraft

In this section, an exact feedback linearization standard form of the dynamics (3) is given by choosing an appropriate diffeomorphic transformation.

Let $x = \begin{bmatrix} x_1^{\mathrm{T}} & x_2^{\mathrm{T}} & \cdots & x_6^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}$, $x_2 = w_s, x_3 = \eta_L, x_4 = \dot{\eta}_L, x_5 = \eta_R, x_6 = \dot{\eta}_R$ and a main body inertia matrix

$$R_s = I_s - FF^{\mathrm{T}}, \tag{10}$$

where x_1 is given in (4), and $F = [F_{sL} \;\; F_{sR}]$. Rewrite the dynamics (3) as follows:

$$\begin{cases} \dot{x} = F(x) + G(x)u, \\ y = h(x), \end{cases} \tag{11}$$

where $F(x) = \begin{bmatrix} f_1^{\mathrm{T}} & f_2^{\mathrm{T}} & \cdots & f_6^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}$, and

$$\begin{aligned} f_1(x) &= C(x_1)x_2, \\ f_2(x) &= R_s^{-1}(-\widetilde{w}_s I_s x_2 + w_L^2 F_{sL} x_3 + 2\xi_L w_L F_{sL} x_4 + w_R^2 F_{sR} x_5 + 2\xi_R w_R F_{sR} x_6), \\ f_3(x) &= x_4, \end{aligned}$$

$$
\begin{aligned}
f_4(x) &= F_{sL}^{\mathrm{T}} R_s^{-1} \widetilde{w}_s I_s x_2 - w_L^2 (I_{l\times l} + F_{sL}^{\mathrm{T}} R_s^{-1} F_{sL}) x_3 - 2\xi_L w_L (I_{l\times l} + F_{sL}^{\mathrm{T}} R_s^{-1} F_{sL}) x_4 \\
&\quad - w_R^2 F_{sL}^{\mathrm{T}} R_s^{-1} F_{sR} x_5 - 2\xi_R w_R F_{sL}^{\mathrm{T}} R_s^{-1} F_{sR} x_6, \\
f_5(x) &= x_6, \\
f_6(x) &= F_{sR}^{\mathrm{T}} R_s^{-1} \widetilde{w}_s I_s x_2 - w_R^2 (I_{l\times l} + F_{sR}^{\mathrm{T}} R_s^{-1} F_{sR}) x_5 - 2\xi_R w_R (I_{l\times l} + F_{sR}^{\mathrm{T}} R_s^{-1} F_{sR}) x_6 \\
&\quad - w_L^2 F_{sR}^{\mathrm{T}} R_s^{-1} F_{sL} x_3 - 2\xi_L w_L F_{sR}^{\mathrm{T}} R_s^{-1} F_{sL} x_4, \\
G(x) &= \left[\begin{array}{cccccc} 0_{3\times 3} & R_s^{-\mathrm{T}} & 0_{3\times l} & -R_s^{-\mathrm{T}} F_{sL} & 0_{3\times l} & -R_s^{-\mathrm{T}} F_{sR} \end{array} \right]^{\mathrm{T}}, \\
u &= T_s, \\
h(x) &= x_1.
\end{aligned}
$$

We first compute the relative degree of (11). We have

$$
\begin{aligned}
L_G h &= 0, \\
L_G L_f h &= C(x_1) R_s^{-1}.
\end{aligned}
$$

If $C(x_1)$ is nonsingular, $L_G L_f h$ is nonsingular, meaning that the relative degree of (11) is $(2, 2, 2)$. From nonlinear control theory we know that (11) can be transformed into an input-output exact feedback linearization form by choosing an appropriate transformation. Choose the following transformation

$$
z = Tx, \tag{12}
$$

where

$$
T = \left[\begin{array}{cccccc}
I_{3\times 3} & 0 & 0 & 0 & 0 & 0 \\
0 & C(x_1) & 0 & 0 & 0 & 0 \\
0 & 0 & I_{l\times l} & 0 & 0 & 0 \\
0 & F_{sL}^{\mathrm{T}} & 0 & I_{l\times l} & 0 & 0 \\
0 & 0 & 0 & 0 & I_{l\times l} & 0 \\
0 & F_{sR}^{\mathrm{T}} & 0 & 0 & 0 & I_{l\times l}
\end{array} \right].
$$

Next, let us prove that Tx is diffeomorphic.

Lemma 3.1 *The nonlinear state transformation in* (12) *for the flexible spacecraft in* (11) *is diffeomophic if* $C(x_1)$ *is nonsingular.*

Proof Compute the Jocobi matrix of (12) as follows:

$$
\frac{\partial (Tx)}{\partial x} = \left[\begin{array}{cccccc}
I_{3\times 3} & 0 & 0 & 0 & 0 & 0 \\
DZ_{21} & C(x_1) & 0 & 0 & 0 & 0 \\
0 & 0 & I_{l\times l} & 0 & 0 & 0 \\
0 & F_{sL}^{\mathrm{T}} & 0 & I_{l\times l} & 0 & 0 \\
0 & 0 & 0 & 0 & I_{l\times l} & 0 \\
0 & F_{sR}^{\mathrm{T}} & 0 & 0 & 0 & I_{l\times l}
\end{array} \right],
$$

where $C(x_1)$ is given in (5), and DZ_{21} is a matrix not used in the following procedure. If $C(x_1)$ is nonsingular, it is easy to see that $\frac{\partial(Tx)}{x}$ is also nonsingular, that is, the transformation (12) is diffeomophic. ∎

In the following, we transform (11) into the exact feedback linearization standard form using the above diffeomopic transformation. Furthermore, let z in (12) be defined as

$$z = \begin{bmatrix} z_1^{\mathrm{T}} & z_2^{\mathrm{T}} & z_3^{\mathrm{T}} & z_4^{\mathrm{T}} & z_5^{\mathrm{T}} & z_6^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}},$$

where z_i, $i = 1, 2, \cdots, 6$ have appropriate dimensions according to the blocks of T. Introduce the following notations according to the form in (2),

$$\xi = \begin{bmatrix} z_1^{\mathrm{T}} & z_2^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}},$$
$$\eta = \begin{bmatrix} z_3^{\mathrm{T}} & z_4^{\mathrm{T}} & z_5^{\mathrm{T}} & z_6^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}.$$

The dynamics (11) can be transformed into the following form by the transformation (12),

$$\begin{aligned} &\dot{z}_1 = z_2, \\ &\dot{z}_2 = -b(z_1)\widetilde{w}_s I_s C(z_1)^{-1} z_2 + A_2(z_1, z_2) z_2 + A_{2\eta}(z_1)\eta + b(z_1)u, \\ &\dot{\eta} = A_\eta \eta + B_\eta(z_1) z_2, \end{aligned} \tag{13}$$

where

$$\begin{aligned} &b(z_1) = C(z_1) R_s^{-1}, \\ &A_2(z_1, z_2) = \frac{\partial C(z_1)}{\partial z_1}(z_2 \otimes I_{3\times 3}) C(z_1)^{-1} - b(z_1)(2\xi_L w_L F_{sL} F_{sL}^{\mathrm{T}} + 2\xi_R w_R F_{sR} F_{sR}^{\mathrm{T}}) C(z_1)^{-1}, \\ &\frac{\partial C(z_1)}{\partial z_1} = \left[\frac{\partial C_{ij}(z_1)}{\partial z_1}\right]_{3\times 9}, i, j = 1, 2, 3, \\ &A_{2\eta}(z_1) = b(z_1)\begin{bmatrix} w_L^2 F_{sL} & 2\xi_L w_L F_{sL} & w_R^2 F_{sR} & 2\xi_R w_R F_{sR} \end{bmatrix}, \\ &A_\eta = \begin{bmatrix} 0 & I_{l\times l} & 0 & 0 \\ -w_L^2 I_{l\times l} & -2\xi_L w_L I_{l\times l} & 0 & 0 \\ 0 & 0 & 0 & I_{l\times l} \\ 0 & 0 & -w_R^2 I_{l\times l} & -2\xi_R w_R I_{l\times l} \end{bmatrix}, \\ &B_\eta(z_1) = \begin{bmatrix} -F_{sL} & 2\xi_L w_L F_{sL} & -F_{sR} & 2\xi_R w_R F_{sR} \end{bmatrix}^{\mathrm{T}} C(z_1)^{-1}. \end{aligned} \tag{14}$$

The conclusion about the relative degree has been proved in the above. Next, we give the properties of the internal dynamics. Above all, we prove that the internal dynamics satisfy the global Lipschitz condition with respect to the external states.

By (13), the internal dynamics is

$$\dot{\eta} = A_\eta \eta + U_\xi, \tag{15}$$

where A_η is a Hurwitz constant matrix. Thus, if we treat U_ξ as an input, the internal dynamics is ISS. By calculation, we have

$$\|f_0(\xi, \eta) - f_0(0, \eta)\| = \|B_\eta(z_1) z_2\| \le L(z)\|\xi\|, \tag{16}$$

where $L(z) = \|B_\eta(z_1)\|$. Next, consider the properties of zero dynamics. By (15), the zero dynamics is

$$\dot{\eta} = A_\eta \eta. \tag{17}$$

It is easy to see that the zero dynamics is a linear system and is exponentially stable.

We summarize the above conclusion into the following lemma.

Lemma 3.2 *The relative degree of the dynamics of the flexible spacecraft* (3) *is* $(2, 2, 2)$. *Furthermore, the zero dynamics in* (17) *is exponentially stable, and the internal dynamics in* (15) *satisfies the local Lipschitz condition and is ISS if* U_ξ *is treated as an input.*

3.2 Control Capability of the Passive Control (7) and (9)

The passive stabilization control in (7) and (9) have been designed for the approximately rigid body external dynamics in (6) without considering flexible components and with small attitudes. In this subsection, we consider the control capability of the passive control (7) and (9), that is, how large flexible components and attitudes the passive controllers can handle.

When considering the control capability, we need the closed-loop dynamics. Substituting (7) into (13), the closed-loop dynamics can be obtained as follows,

$$\begin{aligned} &\dot{z}_1 = z_2, \\ &\dot{z}_2 = A_2(z_1, z_2)z_2 + A_{2\eta}(z_1)\eta + b(z_1)I_s(-K_P x_1 - K_D w_s), \\ &\dot{\eta} = A_\eta \eta + B_\eta(z_1)z_2. \end{aligned} \tag{18}$$

From (18), we can see that large flexible components and attitudes are going to take effect on the rigid body dynamics, that is, the existence of FF^{T} and $C(x_1)$ in the former two equations of the closed-loop dynamics (18) by comparing with the simplified dynamics (6). We will investigate the control capability by analyzing the influence of flexible components and large attitudes on the properties of the rigid body part of the closed-loop dynamics. Under this consideration, we simplify the closed-loop dynamics (18) as follows:

$$\begin{aligned} &\dot{z}_1 = z_2, \\ &\dot{z}_2 = \frac{\partial C(z_1)}{\partial z_1}(z_2 \otimes I_{3\times 3})C(z_1)^{-1}z_2 + b(z_1)I_s(-K_P x_1 - K_D w_s). \end{aligned} \tag{19}$$

We will consider the following two cases in our analysis.

Case 1 The control capability of flexible components.

The existence of the flexible components means that the whole inertia matrix R_s in (10) is not equal to the main body inertia matrix I_s, that is, there exists the flexible term FF^{T}. Let us firstly consider the handling capability of the passive control on the flexible term FF^{T}. Here for simplicity, we consider the cases that $C(x_1) = I$ and hence the closed-loop dynamics (19) becomes

$$\begin{aligned} &\dot{z}_1 = z_2, \\ &\dot{z}_2 = -R_s^{-1}I_s(K_P z_1 + K_D z_2). \end{aligned} \tag{20}$$

Through computation, we have

$$R_s^{-1} = (I_s - FF^{\mathrm{T}})^{-1} = I_s^{-1} + I_s^{-1}F(I_{2l\times 2l} - F^{\mathrm{T}}F)^{-1}F^{\mathrm{T}}I_s^{-1}.$$

Thus,

$$R_s^{-1}I_s = I_{3\times 3} + I_s^{-1}F(I_{2l\times 2l} - F^{\mathrm{T}}F)^{-1}F^{\mathrm{T}}.$$

Denote the second term of the right side of the equal sign of the above equality by a matrix Δ, that is,

$$I_s^{-1}F(I_{2l\times 2l} - F^{\mathrm{T}}F)^{-1}F^{\mathrm{T}} = \Delta,$$

where Δ is a 3 times 3-dimensional matrix, denoted as

$$\Delta = \begin{bmatrix} \Delta_{11} & \Delta_{12} & \Delta_{13} \\ \Delta_{21} & \Delta_{22} & \Delta_{23} \\ \Delta_{31} & \Delta_{32} & \Delta_{33} \end{bmatrix}.$$

Combining (20) and the line and column transformations, we obtain

$$\det\left(sI_{6\times 6} - \begin{bmatrix} 0 & I_{3\times 3} \\ -(I_{3\times 3}+\Delta)K_P & -(I_{3\times 3}+\Delta)K_D \end{bmatrix}\right) = \det\left(\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}\right),$$

where

$$A_{ii} = \begin{bmatrix} s & -1 \\ \overline{\Delta}_{ii}k_{pi} & s + \overline{\Delta}_{ii}k_{di} \end{bmatrix}, \quad \overline{\Delta}_{ii} = 1 + \Delta_{ii}, \quad i = 1,2,3,$$

$$A_{ij} = \begin{bmatrix} 0 & 0 \\ \Delta_{ij}k_{pj} & \Delta_{ij}k_{dj} \end{bmatrix}, \quad i \neq j, \quad i,j = 1,2,3.$$

Here we consider a simple case that $\Delta_{ij} = 0$, $i = 3$, or $j = 3$. According to Routh Criterion, to ensure closed-loop stability, Δ_{ij}, $i = 1,2$, need to satisfy the inequality

$$\Delta_{12}\Delta_{21} - \Delta_{11} - \Delta_{22} - \Delta_{11}\Delta_{22} < 1.$$

From the above inequality, we know that whatever control parameters K_P and K_D are chosen, there exists the flexible term that the passive control cannot handle. Thus, we assumed that FF^{T} is smaller compared with the rigid body inertia matrix I_s, that is, it is assumed that the main body inertia matrix and the whole inertia matrix are approximately the same.

Assumption 2 $R_s = I_s$.

Case 2 The control capability of large attitudes.

We select a simple case where $\theta = 90^{\circ}$ and the other two attitude angles and angular velocities are small. Under this case,

$$C(z_1) = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}.$$

Combining Assumption 2, the closed-loop dynamics (19) becomes

$$\begin{aligned} \dot{z}_1 &= z_2, \\ \dot{z}_2 &= -C(z_1)(K_P z_1 + K_D w_s). \end{aligned} \tag{21}$$

The characteristic equations of the closed-loop system (21) can be given as

$$\begin{aligned} &s^4 + (k_{d1} + k_{d3})s^3 + k_{d1}k_{d3}s^2 + k_{p1}k_{p3} = 0, \\ &s^2 + k_{d2}s + k_{p2} = 0, \end{aligned}$$

which can be seen with unstable eigenvalues after simple analysis. By the above consideration, we conclude that, in general, the control laws in (7) is only feasible for the case when the attitudes are small. Thus, without loss of generality, we may assume

Assumption 3 $C(x_1) = I_{3\times 3}$.

Remark 3.3 The passive PD control (7) and (9) can only be used for the cases that the Euler angles are small, and that the main body inertia matrix I_s and the whole inertia matrix R_s are approximately the same.

3.3 Properties of the Passive Control (7) and (9)

In this subsection we study the closed-loop properties of the flexible spacecraft dynamics.

Under Assumptions 2 and 3, the closed-loop dynamics (18) becomes

$$\begin{aligned} \dot{z}_1 &= z_2, \\ \dot{z}_2 &= -K_P z_1 - K_D z_2 + \Psi(z), \\ \dot{\eta} &= A_\eta \eta + B_\eta z_2, \end{aligned} \tag{22}$$

where

$$\begin{aligned} \Psi(z) &= -I_s^{-1}(2\xi_L w_L F_{sL} F_{sL}^{\mathrm{T}} + 2\xi_R w_R F_{sR} F_{sR}^{\mathrm{T}})z_2 + I_s^{-1} a_\eta \eta, \\ a_\eta &= \begin{bmatrix} w_L^2 F_{sL} & 2\xi_L w_L F_{sL} & w_R^2 F_{sR} & 2\xi_R w_R F_{sR} \end{bmatrix}, \end{aligned}$$

B_η is given in (14), and $\Psi(z)$ and B_η are the coupling terms between rigid and flexible bodies.

The following theorem shows the properties of the closed-loop dynamics.

Theorem 3.4 *Suppose that Assumptions* 2 *and* 3 *hold. For the flexible dynamics* (3)*, design the passive stabilization control in* (7)*. Then the closed-loop system is locally exponential stability including the flexible states by adjusting the control parameters sufficiently large.*

Proof By Lemma 3.2, the zero dynamics (17) is exponentially stable. By the inverse Lyapunov theorem, in some region of $\eta = 0$, there exists a Lyapunov function $V_1(\eta)$, such that

$$c_1\|\eta\|^2 \le V_1(\eta) \le c_2\|\eta\|^2,$$

$$\frac{\partial V_1(\eta)}{\partial \eta} f_0(0,\eta) \le -c_3\|\eta\|^2,$$

$$\left\| \frac{\partial V_1(\eta)}{\partial \eta} \right\|_2 \le c_4\|\eta\|.$$

and

$$\|\Psi\| \le k(\|z_2\| + \|\eta\|). \tag{23}$$

Next, we analyze the stability of the closed-loop system. Choosing a Lyapunov function

$$V = z_1^{\mathrm{T}} K_P z_1 + z_2^{\mathrm{T}} z_2 + V_1(\eta)$$

and combining (16) and (23), it can be derived that

$$\dot{V} \le [\|z_2\| \ \ \|\eta\|] \begin{bmatrix} -2\lambda_{\min}(K_D) + 2k & k + Lc_4/2 \\ k + Lc_4/2 & -c_3 \end{bmatrix} \begin{bmatrix} \|z_2\| \\ |\eta\| \end{bmatrix}.$$

Then it is obtained that $\dot{V} \le 0$ is equivalent to

$$\lambda_{\min}(K_D) > \frac{(k + Lc_4/2)^2}{c_3} + k.$$

Furthermore, by the Lasalle invariant set theory, when K_D is large enough, the system is locally stable. ∎

Remark 3.5 By Theorem 3.4, the flexible modes can decay by themselves.

Remark 3.6 Theorem 3.4 is different from Lemma 13.4 in [8], where the uncertainties are assumed small enough in Lemma 13.4 (k is small enough), which implies that the designed feedback linearization control is robust for the sufficiently small uncertainties, while this paper shows that the designed control has a attenuation ability for existing uncertainties without any assumption.

Furthermore, we consider the properties of passive tracking control under Assumptions 1–3. Substituting (9) into (13), we can obtain the closed-loop system as follows,

$$\begin{aligned} \dot{z}_1 &= z_2, \\ \dot{z}_2 &= -K_P z_1 - K_D z_2 + \Psi(z) + \ddot{r} + K_D \dot{r} + K_P r, \\ \dot{\eta} &= A_\eta \eta + B_\eta z_2. \end{aligned} \tag{24}$$

By comparing with (22) the homogeneous component of (24) is the same as that of (22). Thus, taking advantage of the proof method of Theorem 3.4, it can be proved that the homogeneous component of (24) is locally exponentially stable. Furthermore, because the homogeneous component is linear, (24) is BIBS. By Assumption 1, we can have the following theorem.

Theorem 3.7 *Suppose that Assumptions* 1–3 *hold. For the flexible spacecraft in* (3)*, design the passive tracking control in* (9)*. Then, the flexible modes and the tracking errors are bounded by adjusting the parameters of* (9) *sufficiently large.*

Rewrite (24) as the form of errors

$$\ddot{e} + K_D\dot{e} + K_P e = \overline{\Psi} - I_s^{-1}(2\xi_L w_L F_{sL} F_{sL}^{\mathrm{T}} + 2\xi_R w_R F_{sR} F_{sR}^{\mathrm{T}})\dot{r}, \tag{25}$$

where

$$\overline{\Psi}(z,e) = -I_s^{-1}(2\xi_L w_L F_{sL} F_{sL}^{\mathrm{T}} + 2\xi_R w_R F_{sR} F_{sR}^{\mathrm{T}})\dot{e} + I_s^{-1} a_\eta \eta,$$
$$e = z_1 - r.$$

When $\dot{r} = 0$, (25) becomes

$$\ddot{e} + K_D\dot{e} + K_P e = \overline{\Psi}.$$

Combining the last equation in (24) and treating $[e \ \dot{e} \ \eta]^{\mathrm{T}}$ as states, and taking advantage of the same proof procedure as in Theorem 3.4, we can prove the following theorem.

Theorem 3.8 *Suppose that Assumptions* 1–3 *hold, and the tracking references are constant. For the flexible spacecraft in* (3)*, design the passive tracking control in* (9)*. Then the flexible modes and the tracking errors are exponentially stable if the control parameters are sufficiently large.*

Remark 3.9 By Theorem 3.7, the coupled term between rigid and flexible modes

$$-I_s^{-1}(2\xi_L w_L F_{sL} F_{sL}^{\mathrm{T}} + 2\xi_R w_R F_{sR} F_{sR}^{\mathrm{T}})\dot{r}$$

leads to tracking errors of the closed-loop system. In practice, an integral control is usually introduced to cancel steady errors, but only constant errors. Hence, the tracking errors cannot be completely cancelled by designing an integral control.

4 Conclusions

The attenuation ability of passive PD control in the context of practical flexible spacecraft is investigated. We show that PD control can achieve local stabilization for flexible spacecraft with flexible modes, but the steady error is inevitable for tracking problems since the coupling between rigid and flexible modes cannot be cancelled completely by additionally designing an integral control. It is suggested that advanced adaptive control methods should be introduced for the tracking control of flexible spacecraft instead of PID control. Such an investigation lays an important foundation for the practical applications of flexible spacecraft.

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